

On the suppression of the $n\bar{n}$ transitions in medium

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Abstract

The new model of $n\bar{n}$ transitions in the medium based on unitary S -matrix is considered. The time-dependence and corrections to the model are studied. The lower limit on the free-space $n\bar{n}$ oscillation time τ is in the range $10^{16} \text{ yr} > \tau > 1.2 \cdot 10^9 \text{ s}$.

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1 Introduction

In the standard calculations of ab oscillations in the medium [1,2] the interaction of particles a and b with the matter is described by the potentials $U_{a,b}$ (potential model). $\text{Im}U_b$ is responsible for loss of b -particle intensity. We consider the $n\bar{n}$ transitions [3,4] in a medium followed by annihilation:

$$n \rightarrow \bar{n} \rightarrow M, \quad (1)$$

where M are the annihilation mesons. For the process (1) potential model is used as well [5-12].

In [10,11] it was shown that one-particle (potential) model mentioned above does not describe the process (1) and thus total neutron-antineutron transition probability: the process (1) probability is $W \sim \Gamma$ (see (15)), whereas the potential model gives $W \sim 1/\Gamma$, where Γ is the annihilation width of \bar{n} in the medium (see eq. (6) of Ref. [5] or eq. (1.6) of Ref. [9] or eq. (16) of present paper). In the potential model the effect of final state absorption (annihilation) acts in the *opposite* (wrong) direction, which tends to the additional *suppression* of the $n\bar{n}$ transition. Since the annihilation is the main effect which defines the speed of process (1), the potential model should be rejected. This is because the unitarity condition is used for the essentially non-unitary S -matrix [10,11]. The interaction Hamiltonian contains the antineutron optical potential $U_{\bar{n}}$ and $\text{Im}U_{\bar{n}}$ plays a crucial role. The S -matrix should be *unitary*.

More formally, the basic equation $\sum_{f \neq i} |T_{fi}|^2 \approx 2\text{Im}T_{ii}$, $S = 1 + iT$ follows from the unitarity condition $(SS^+)_{fi} = \delta_{fi}$. However in the potential model the S -matrix is *essentially* non-unitary $(SS^+)_{fi} = \delta_{fi} + \alpha_{fi}$, resulting in $\sum_{f \neq i} |T_{fi}|^2 \approx 2\text{Im}T_{ii} + \alpha_{ii} \neq 2\text{Im}T_{ii}$ because $2\text{Im}T_{ii}$ is extremely small: $2\text{Im}T_{ii} < 10^{-31}$ [10,11]. The above-given basic equation is inapplicable in this case.

The potential model was developed in [5-8]. In more recent papers the various details of the model have been refined, in particular the parameters of optical potential. We don't dwell on these papers since the heart of the problem is in the non-Hermiticity of the optical potential.

In [13] we have proposed the model based on the diagram technique for direct reactions which does not contain the non-Hermitian operators. Subsequently, this calculation was repeated in [14,15]. However, in [16] it was shown that this model is unsuitable because the $n\bar{n}$ transition takes place in the propagator which is wrong. The neutron of the nucleus is in the bound state and so the neutron line entering into the $n\bar{n}$ transition vertex should be the wave function, but not the propagator, as in the model based on the diagram technique. For the problem under study this fact is crucial. It leads to the cardinal error for the process in nuclei. The $n\bar{n}$ transitions in the medium and vacuum are not reproduced at all. If the neutron binding energy goes to zero, the result diverges (see eqs. (18) and (19) of ref. [13] or eqs. (15) and (17) of ref. [14]). So we abandoned this model [16] (for more details, see [17]).

In [10] the model which is free of drawbacks given above has been proposed (model **b** in the notations of present paper). However, the consideration was schematic since our concern was only with the role of the final state absorption in principle. In sect. 2 this model as well as the model with bare propagator are studied in detail. The corrections to the models (sect. 3) and time-dependence (sect. 4) are considered as well. In sect. 5 we sum up the present state of the investigations of the $n\bar{n}$ transition problem.

Also we want to attract the attention to the problem under consideration because of the following reasons: 1) For the lower limit on the free-space $n\bar{n}$ oscillation time the range of uncertainty is too wide (see eq. (38)). 2) The problem involves a number of questions which are of independent interest.

The basic material is given in sects. 2 and 5. Figure 1 and analysis of amplitudes M_a and M_b are of major importance.

2 Models

First of all we consider the antineutron annihilation in the medium. The annihilation amplitude M_a is defined as

$$\langle f0 | T \exp(-i \int dx \mathcal{H}(x)) - 1 | 0\bar{n}_p \rangle = N(2\pi)^4 \delta^4(p_f - p_i) M_a. \quad (2)$$

Here \mathcal{H} is the Hamiltonian of the \bar{n} -medium interaction, $|0\bar{n}_p\rangle$ is the state of the medium containing the \bar{n} with the 4-momentum $p = (\epsilon, \mathbf{p})$; $\langle f |$ denotes the annihilation products, N includes the normalization factors of the wave functions. The antineutron annihilation width Γ is expressed through M_a :

$$\Gamma \sim \int d\Phi |M_a|^2. \quad (3)$$

For the Hamiltonian \mathcal{H} we consider the model

$$\begin{aligned} \mathcal{H} &= \mathcal{H}_a + V \bar{\Psi}_{\bar{n}} \Psi_{\bar{n}}, \\ H(t) &= \int d^3x \mathcal{H}(x) = H_a(t) + V, \end{aligned} \quad (4)$$

where \mathcal{H}_a is the effective annihilation Hamiltonian in the second quantization representation, V is the residual scalar field. The diagrams for the model (4) are shown in fig. 1. The first diagram corresponds to the first order in \mathcal{H}_a and so on.

Consider now the process (1). The qualitative process picture is as follows. The free-space $n\bar{n}$ transition comes from the exchange of Higgs bosons with the mass $m_H > 10^5$ GeV [4] and so the subprocess of $n\bar{n}$ conversion is scarcely affected by a medium effects. From the dynamical point of view this is a momentary process: $\tau_c \sim 1/m_H < 10^{-29}$ s. The antineutron annihilates in a time $\tau_a \sim 1/\Gamma$. We deal with two-step process with the characteristic time $\tau_{ch} \sim \tau_a$.

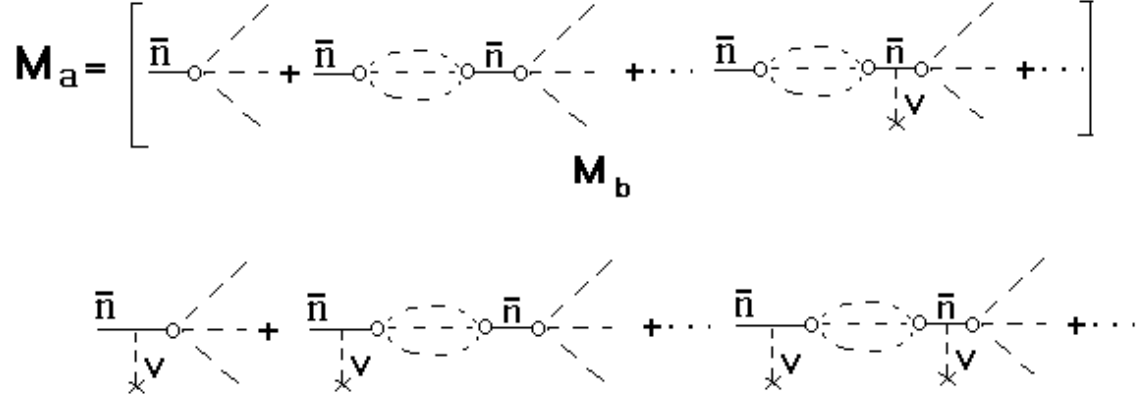


Figure 1: Antineutron annihilation in the medium. The annihilation is shown by a circle

The neutron wave function is

$$n(x) = \Omega^{-1/2} \exp(-ipx). \quad (5)$$

Here $p = (\epsilon, \mathbf{p})$ is the neutron 4-momentum; $\epsilon = \mathbf{p}^2/2m + U_n$, where U_n is the neutron potential. The interaction Hamiltonian has the form

$$H_I = H_{n\bar{n}} + H, \quad (6)$$

$$H_{n\bar{n}}(t) = \int d^3x (\epsilon_{n\bar{n}} \bar{\Psi}_{\bar{n}}(x) \Psi_n(x) + H.c.) \quad (7)$$

Here $H_{n\bar{n}}$ is the Hamiltonian of $n\bar{n}$ conversion [6], $\epsilon_{n\bar{n}}$ is a small parameter with $\epsilon_{n\bar{n}} = 1/\tau_{n\bar{n}}$, where $\tau_{n\bar{n}}$ is the free-space $n\bar{n}$ oscillation time; $m_n = m_{\bar{n}} = m$. In the lowest order in $H_{n\bar{n}}$ the amplitude of process (1) is *uniquely* determined by the Hamiltonian (6):

$$M = \epsilon_{n\bar{n}} G_0 M_a, \quad (8)$$

$$G_0 = \frac{1}{\epsilon - \mathbf{p}^2/2m - U_n + i0}, \quad (9)$$

$\mathbf{p}_{\bar{n}} = \mathbf{p}$, $\epsilon_{\bar{n}} = \epsilon$. Here G_0 is the antineutron propagator. The corresponding diagram is shown in fig. 2a. The annihilation amplitude M_a is given by (2), where $\mathcal{H} = \mathcal{H}_a + V \bar{\Psi}_{\bar{n}} \Psi_{\bar{n}}$. Since M_a contains all the \bar{n} -medium interactions followed by annihilation including antineutron rescattering in the initial state, the antineutron propagator G_0 is bare. Once the antineutron annihilation amplitude is defined by (2), the expression for the process amplitude (8) *rigorously follows* from (6). For the time being we do not go into the singularity $G_0 \sim 1/0$.

One can construct the model with the dressed propagator. We include the scalar field V in the antineutron Green function

$$G_d = G_0 + G_0 V G_0 + \dots = \frac{1}{(1/G_0) - V} = -\frac{1}{V} = -\frac{1}{\Sigma}, \quad (10)$$

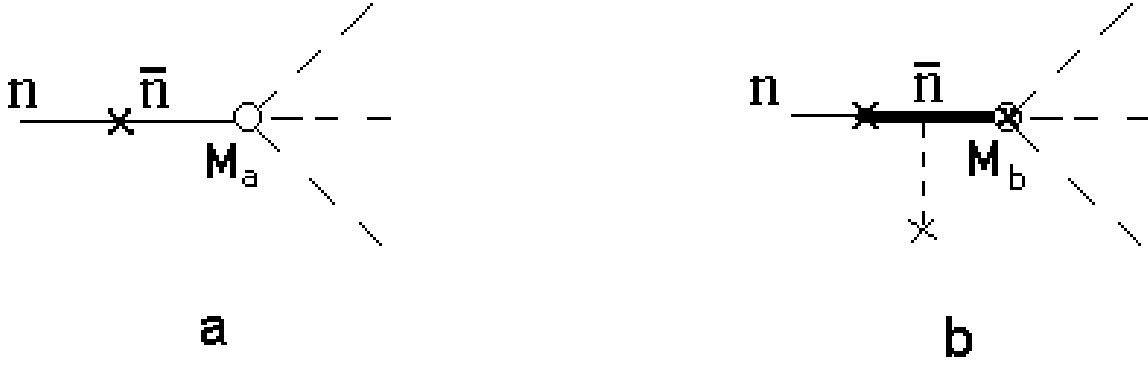


Figure 2: **a** $n\bar{n}$ transition in the medium followed by annihilation. The antineutron annihilation is shown by a circle. **b** Same as **a** but the antineutron propagator is dressed (see text)

$\Sigma = V$, where Σ is the antineutron self-energy. The process amplitude is

$$M = \epsilon_{n\bar{n}} G_d M_b, \quad (11)$$

$G_d M_b = G_0 M_a$ (see fig. 2b). The block in the square braces shown in fig.1 corresponds to the vertex function M_b . The models shown in figs. 2a and 2b we denote as the models **a** and **b**, respectively.

In both models the antineutron propagators don't contain the annihilation loops since the annihilation is taken into account in the amplitudes M_a and M_b ; the interaction Hamiltonians H_I and unperturbed Hamiltonians are the same. If $\Sigma \rightarrow 0$, the model **b** goes into model **a**. In this sense the model **a** is the limiting case of the model **b**.

We consider the model **b**. For the process width Γ_b one obtains

$$\Gamma_b = N_1 \int d\Phi |M|^2 = \frac{\epsilon_{n\bar{n}}^2}{\Sigma^2} N_1 \int d\Phi |M_b|^2 = \frac{\epsilon_{n\bar{n}}^2}{\Sigma^2} \Gamma', \quad (12)$$

$$\Gamma' = N_1 \int d\Phi |M_b|^2, \quad (13)$$

where Γ' is the annihilation width of \bar{n} calculated through the M_b (and not M_a). The normalization multiplier N_1 is the same for Γ_b and Γ' . The vertex function M_b is unknown. (We recall the antineutron annihilation width Γ is expressed through the amplitude M_a .) For the estimation we put

$$M_b = M_a, \quad \Gamma' = \Gamma. \quad (14)$$

This is an uncontrollable approximation.

The time-dependence is determined by the exponential decay law:

$$W_b(t) = 1 - e^{-\Gamma_b t} \approx \frac{\epsilon_{n\bar{n}}^2}{\Sigma^2} \Gamma' t = \frac{\epsilon_{n\bar{n}}^2}{\Sigma^2} \Gamma t. \quad (15)$$

Equation (15) illustrates the result sensitivity to the value of parameter Σ .

On the other hand, for $n\bar{n}$ transitions in nuclear matter the potential model gives the inverse Γ -dependence [6-11]

$$W_{\text{pot}}(t) = 2\epsilon_{n\bar{n}}^2 t \frac{\Gamma/2}{(\text{Re}U_{\bar{n}} - U_n)^2 + (\Gamma/2)^2} \approx \frac{4\epsilon_{n\bar{n}}^2 t}{\Gamma}, \quad (16)$$

where $U_{\bar{n}}$ is the antineutron optical potential. The wrong Γ -dependence is a direct consequence of the inapplicability of the model based on optical potential for the calculation of the total process probability [11]. (The above-mentioned model describes the probability of finding an antineutron only.)

Comparing with (15), one obtains

$$r = \frac{W_b}{W_{\text{pot}}} = \frac{\Gamma^2}{4\Sigma^2} = 25, \quad (17)$$

where the values $\Gamma = 100$ MeV and $\Sigma = 10$ MeV have been used. The parameter Σ is uncertain. We have put $\Sigma = \text{Re}U_{\bar{n}} - U_n \approx 10$ MeV only for estimation.

The model **b** leads to an increase of the $n\bar{n}$ transition probability. The lower limit on the free-space $n\bar{n}$ oscillation time τ^b increases as well. Let τ^b and τ_{pot} be the lower limits on the free-space $n\bar{n}$ oscillation time obtained by means of eqs. (15) and (16), respectively; $T_{n\bar{n}}$ is the oscillation time of neutron bound in a nucleus. The relationships between τ^b , τ_{pot} and $T_{n\bar{n}}$ are

$$\tau^b = \sqrt{r}\tau_{\text{pot}} = \frac{\Gamma}{2\Sigma}\tau_{\text{pot}}, \quad (18)$$

$$\tau^b = \frac{1}{\Sigma}\sqrt{\Gamma T_{n\bar{n}}}, \quad (19)$$

where the well-known equation

$$\tau_{\text{pot}} = 2\sqrt{T_{n\bar{n}}/\Gamma} \quad (20)$$

has been used. For estimation we take $\Gamma = 100$ MeV, $\Sigma = 10$ MeV and $\tau_{\text{pot}} = 2.36 \cdot 10^8$ s [18]. (The limit $\tau_{\text{pot}} = 2.36 \cdot 10^8$ s was derived [18] through eq. (20) from experimental bound on the neutron lifetime in oxygen $T_{n\bar{n}} > 1.77 \cdot 10^{32}$ yr obtained by Super-Kamiokande collaboration [18].) Equation (18) gives

$$\tau^b = 5\tau_{\text{pot}} = 1.2 \cdot 10^9 \text{ s}. \quad (21)$$

If $\Sigma \rightarrow 0$, eq. (18) diverges: $\tau^b \rightarrow \infty$. This circumstance should be clarified; otherwise the model under consideration can be rejected. As we will see later, the correct formulation of the problem (on the finite time interval) leads to finite result for the model **a**, which justifies our approach.

We return to the model shown in fig. 2a. We use the basis (n, \bar{n}) . The results do not depend on the basis. A main part of existing calculations have been done in $n - \bar{n}$ representation.

The physics of the problem is in the Hamiltonian. The transition to the basis of stationary states is a formal step. It has a sense only in the case of the potential model $H = H_{\text{pot}} = \text{Re}U_{\bar{n}} - U_n - i\Gamma/2 = \text{const.}$, when the Hamiltonian of \bar{n} -medium interaction is replaced by the effective mass $H \rightarrow H_{\text{pot}} = m_{\text{eff}}$ because the Hermitian Hamiltonian of interaction of the stationary states with the medium is unknown. Since we work beyond the potential model, the procedure of diagonalization of mass matrix is unrelated to our problem.

The amplitude (8) diverges

$$M = \epsilon_{n\bar{n}} G_0 M_a \sim \frac{1}{0}. \quad (22)$$

(See also eq. (21) of ref. [16].) These are infrared singularities conditioned by zero momentum transfer in the $n\bar{n}$ transition vertex. (In the model **b** the effective momentum transfer $q_0 = V = \Sigma$ takes place.)

For solving the problem the field-theoretical approach with finite time interval [19] is used. It is infrared free. If $H = H_{\text{pot}}$, the approach with finite time interval reproduces all the results on the particle oscillations (see sect. 5.2 of ref. [12]). This is the test of above-mentioned approach. However, our purpose is to describe the process (1) by means of Hermitian Hamiltonian because the potential model describes the absorption wrongly.

For the model **a** the process (1) probability was found to be [12,16]

$$W_a(t) \approx W_f(t) = \epsilon_{n\bar{n}}^2 t^2, \quad \Gamma t \gg 1, \quad (23)$$

where W_f is the free-space $n\bar{n}$ transition probability. Owing to annihilation channel, W_a is practically equal to the free-space $n\bar{n}$ transition probability. If $t \rightarrow \infty$, eq. (23) diverges just as the modulus (22) squared does. If $\Sigma \rightarrow 0$, eq. (15) diverges quadratically as well.

The explanation of the t^2 -dependence is simple. The process shown in fig. 2a represents two consecutive subprocesses. The speed and probability of the whole process are defined by those of the slower subprocess. If $1/\Gamma \ll t$, the annihilation can be considered instantaneous. So, the probability of process (1) is defined by the speed of the $n\bar{n}$ transition: $W_a \approx W_f \sim t^2$.

Distribution (23) leads to very strong restriction on the free-space $n\bar{n}$ oscillation time [12,16]:

$$\tau^a = 10^{16} \text{ yr}. \quad (24)$$

If $\Sigma \rightarrow 0$, W_b rises quadratically. So τ^b and τ^a can be considered as the estimations from below and above, respectively.

3 Corrections

We show that for the $n\bar{n}$ transition in medium the corrections to the models and additional baryon-number-violating processes (see fig. 3) cannot essentially change the results. First of

all we consider the incoherent contribution of the diagrams 3. In fig. 3a a meson is radiated before the $n\bar{n}$ transition. The interaction Hamiltonian has the form

$$H_I = \int d^3x g \Psi_n^+ \Phi \Psi_n + H_{n\bar{n}} + H. \quad (25)$$

In the following the background neutron potential is omitted. The neutron wave function is given by (5), where $p = (p_0, \mathbf{p})$ and $p_0 = m + \mathbf{p}^2/2m$.

For the process amplitude M_{3a} one obtains

$$M_{3a} = gG\epsilon_{n\bar{n}}GM^{(n-1)}, \quad (26)$$

$$G = \frac{1}{p_0 - q_0 - m - (\mathbf{p} - \mathbf{q})^2/2m + i0}, \quad (27)$$

where q is the 4-momentum of meson radiated, $M^{(n-1)}$ is the amplitude of antineutron annihilation in the medium in the $(n-1)$ mesons. As with model **a**, the antineutron propagator G is *bare*; the \bar{n} self-energy $\Sigma = 0$. (The same is true for figs. 3b-3d.)

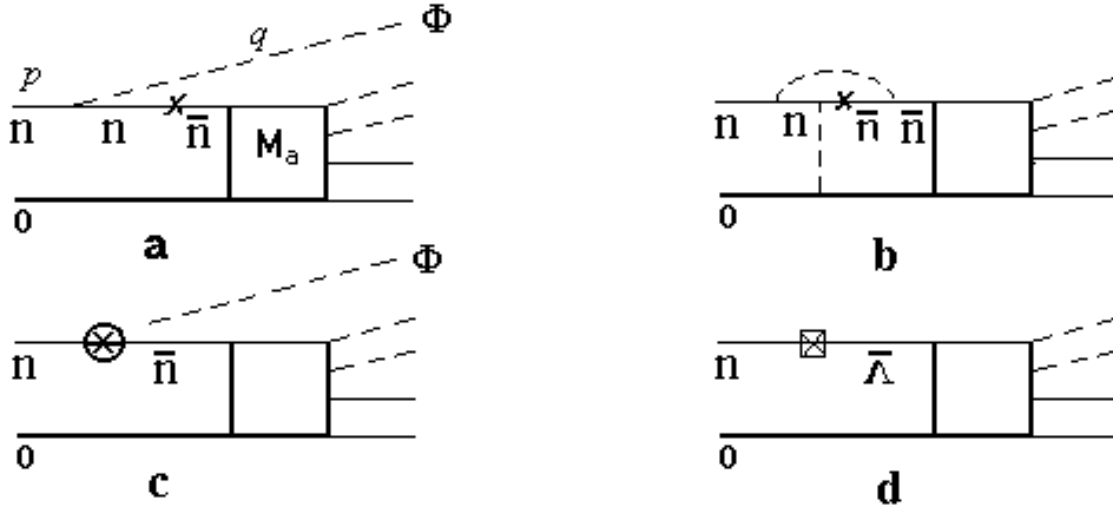


Figure 3: Corrections to the models (**a** and **b**) and additional baryon-number-violating processes (**c** and **d**)

If $q \rightarrow 0$, the amplitude M_{3a} increases since $G \rightarrow G_s$,

$$G_s = \frac{1}{p_0 - m - \mathbf{p}^2/2m} \sim \frac{1}{0}. \quad (28)$$

(The limiting transition $q \rightarrow 0$ for the diagram 3a is an imaginary procedure because in the vertex $n \rightarrow n\Phi$ the real meson is escaped and so $q_0 \geq m_\Phi$.) The fact that the amplitude increases is essential for us because for fig. 2a $q = 0$. Due to this $G_0 \sim 1/0$ and $W_a \gg W_b$.

Let Γ_{3a} and $\Gamma^{(n)}$ be the widths corresponding to the fig. 3a and annihilation width of \bar{n} in the (n) mesons, respectively; $\Gamma = \sum_{(n)} \Gamma^{(n)}$. Taking into account that $\Gamma^{(n)}$ is a smooth function of \sqrt{s} and summing over (n) , it is easy to get the estimation:

$$\Gamma_{3a} \approx 5 \cdot 10^{-3} g^2 \frac{\epsilon_{n\bar{n}}^2}{m_\Phi^2} \Gamma \approx \frac{\epsilon_{n\bar{n}}^2}{m_\Phi^2} \Gamma. \quad (29)$$

The time-dependence is determined by the exponential decay law:

$$W_{3a}(t) \approx \Gamma_{3a} t = \frac{\epsilon_{n\bar{n}}^2}{m_\Phi^2} \Gamma t. \quad (30)$$

Comparing with (15) we have: $W_{3a}/W_b = V^2/m_\Phi^2 \ll 1$. So for the model **b** the contribution of diagram 3a is negligible.

For the model **a** the contribution of diagram 3a is inessential as well. Indeed, using eqs. (30) and (23) we get

$$\frac{W_{3a}(t)}{W_a(t)} = \frac{\Gamma}{m_\pi^2 t}, \quad (31)$$

where we have put $m_\Phi = m_\pi$. Consequently, if

$$m_\pi^2 t / \Gamma \gg 1, \quad (32)$$

and

$$\Gamma t \gg 1 \quad (33)$$

(see (23)) then the contribution of diagram 3a is negligible. For the $n\bar{n}$ transition in nuclei these conditions are fulfilled since in this case $\Gamma \sim 100$ MeV and $t = T_0 = 1.3$ yr, where T_0 is the observation time in proton-decay type experiment [18]. In fact, it is suffice to hold condition (33) only because it is more strong.

In the calculations made above the free-space $n\bar{n}$ transition operator has been used. This is impulse approximation which is employed for nuclear β decay, for instance. The simplest medium correction to the vertex (or off-diagonal mass, or transition mass) is shown in fig. 3b. In this event the replacement should be made:

$$\epsilon_{n\bar{n}} \rightarrow \epsilon_m = \epsilon_{n\bar{n}}(1 + \Delta\epsilon), \quad (34)$$

$\Delta\epsilon = \epsilon_{3b}/\epsilon_{n\bar{n}}$, where ϵ_{3b} is the correction to $\epsilon_{n\bar{n}}$ produced by the diagram 3b. For the model **a** the limit becomes

$$\tau^a = (1 + \Delta\epsilon) 10^{16} \text{ yr}. \quad (35)$$

Obviously, the $\Delta\epsilon$ cannot change the order of magnitude of τ^a since the $n \rightarrow \bar{n}$ operator is essentially zero-range one. The free-space $n\bar{n}$ transition comes from the exchange of Higgs bosons with the mass $m_H > 10^5$ GeV [4]. Since $m_H \gg m_W$ (m_W is the mass of W -boson), the

renormalization effects should not exceed those characteristic of nuclear β decay which is less than 0.25 [20]. So the medium corrections to the vertex are inessential for us. The same is true for the model **b**.

Consider now the baryon-number-violating decay $n \rightarrow \bar{n}\Phi$ [21] shown in fig. 3c. It leads to the same final state, as the processes depicted in figs. 2, 3a and 3b. Denoting $|\mathbf{q}| = q$, for the decay width Γ_{3c} one obtains

$$\Gamma_{3c} = \frac{\epsilon_\Phi^2}{(2\pi)^2} \int dq \frac{q^2}{q_0} G^2 \Gamma(q), \quad (36)$$

$q_0^2 = q^2 + m_\Phi^2$. The parameter ϵ_Φ corresponding to the vertex $n \rightarrow \bar{n}\Phi$ is unknown and so no detailed calculation is possible.

The baryon-number-violating conversion $n \rightarrow \bar{\Lambda}$ in the medium [21] shown in fig. 3d cannot produce interference, since it contains K -meson in the final state. For the rest of the diagrams the significant interferences are unlikely because the final states in $\bar{n}N$ annihilation are very complicated configurations and persistent phase relations between different amplitudes cannot be expected. This qualitative picture is confirmed by our calculations [22] for \bar{p} -nuclear annihilation. It is easy to verify the following statement: if the incoherent contribution of the diagrams 3a-3c to the total nuclear annihilation width is taken into account, the lower limit on the free-space $n\bar{n}$ oscillation time τ becomes even better.

To summarise, the contribution of diagrams 3 is inessential for us.

4 Time-dependence

The non-trivial circumstance is the quadratic time-dependence in the model **a**: $W_a \sim t^2$. The heart of the problem is as follows. The processes depicted by the diagrams 2b and 3 are described by the exponential decay law. In the first vertex of these diagrams the momentum transfer (figs. 3a-3c), or effective momentum transfer (figs. 2b, 3d) takes place. The diagram 2a contains the infrared divergence conditioned by zero momentum transfer in the $n\bar{n}$ transition vertex. This is unremovable peculiarity. This means that the standard S -matrix approach is inapplicable [12,16,19]. In such an event, the other surprises can be expected as well. From this standpoint a non-exponential behaviour comes as no surprise to us. It seems natural that for non-singular and singular diagrams the functional structure of the results is different, including the time-dependence. The opposite situation would be strange.

The fact that for the processes with $q = 0$ the S -matrix problem formulation $(\infty, -\infty)$ is physically incorrect can be seen even from the limiting case $H = 0$: if $H_I = H_{n\bar{n}}$ (see (6)), the solution is periodic. It is obtained by means of non-stationary equations of motion and not S -matrix theory. To reproduce the limiting case $H \rightarrow 0$, i.e. the periodic solution, we have to

use the approach with finite time interval.

If the problem is formulated on the interval $(t, 0)$, the decay width Γ cannot be introduced since $\Gamma = \sum_{f \neq i} |S_{fi}(\infty, -\infty)|^2 / T_0$, $T_0 \rightarrow \infty$. This means that the standard calculation scheme should be completely revised. (We would like to emphasise this fact.) The direct calculation by means of evolution operator gives the distribution (23).

Formally, the different time-dependence is due to q -dependence of amplitudes. We consider eq. (26), for instance. If q decreases, the amplitude M_{3a} increase; in the limit $q \rightarrow 0$ it is singular (see (28)). The point $q = 0$ corresponds to realistic process shown in fig. 2a. The t^2 -dependence of this process is the consequence of the zero momentum transfer.

The more physical explanation of the t^2 -dependence is as follows. In the Hamiltonian (25) corresponding to fig. 3a we put $H = H_{n\bar{n}} = 0$. Then the virtual decay $n \rightarrow n\Phi$ takes place. The first vertex of the diagram 3a dictates the exponential decay law of the overall process shown in fig. 3a. Similarly, in the Hamiltonian (6) corresponding to fig. 2a, we put $H = 0$. Then the free-space $n\bar{n}$ transition takes place which is quadratic in time: $W_f(t) = \epsilon_{n\bar{n}}^2 t^2$. The first vertex determines the time-dependence of the whole process at least for small Γ . We also recall that even for proton decay the possibility of non-exponential behaviour is realistic [23-25].

5 Discussion and summary

In both models the antineutron propagators don't contain the annihilation loops since the annihilation is taken into account in the amplitudes M_a and M_b . The alternative model containing the full in-medium propagator has been considered in [26]. The sole physical distinction between models **a** and **b** is the definition of antineutron annihilation amplitude; or, similarly, the non-zero antineutron self-energy in the model **b** which is conditioned by residual scalar field. However, it leads to the fundamentally different results.

If $\Sigma \rightarrow 0$, $W_b(t)$ diverges quadratically. This circumstance should be clarified; otherwise the model **b** can be rejected. The calculation in the framework of the model **a** gives the finite result, which justifies our approach from a conceptual point of view and consideration of the model **a** at least as the limiting case. In fact, the model **a** seems quite realistic in itself. Indeed, we list the main drawbacks of the model **b**.

1) The approximation $M_b = M_a$ is an uncontrollable one. The value of Σ is uncertain. These points are closely related.

2) The diagram 2b means that the annihilation is turned on upon forming of the self-energy part $\Sigma = V$ (after multiple rescattering of \bar{n}). This is counter-intuitive since at low energies [27,28]

$$\sigma_a > 2.5\sigma_s, \quad (37)$$

where σ_a and σ_s are the cross sections of free-space $\bar{n}N$ annihilation and $\bar{n}N$ scattering, respectively. The inverse picture is in order: in the first stage of the \bar{n} -medium interaction the annihilation occurs. This is obvious for the $n\bar{n}$ transitions in the gas. The model **a** reproduces the *competition* between scattering and annihilation in the intermediate state [26].

3) The time-dependence is a more important characteristic of any process. It is common knowledge that the t -dependence of the process probability in the vacuum and medium is identical (for example, exponential decay law (15)). In the model **a** the t -dependencies in the vacuum and medium coincide: $W_a \sim t^2$ and $W_f \sim t^2$. The model **b** gives $W_b \sim t$, whereas $W_f \sim t^2$. There is no reason known why we have such a fundamental change.

4) If $H = U_{\bar{n}}$, the model **a** reproduces all the well-known results on particle oscillations in contrast to the model **b** (see sect. 5.2 of ref. [12]). In other words, the model **a** reproduces all the results of potential model. (Recall that our purpose is to describe the process (1) by means of Hermitian Hamiltonian because the potential model describes the absorption wrongly.)

The model **a** is free of drawbacks given above. The physics of the model is absolutely standard. For instance, for the processes shown in fig. 3 the antineutron propagators are bare as well. The same is also true for the diagram technique for direct reactions [13-15]. Besides, the process amplitude (8) is derived *directly* from the basic eqs. (6) and (2) which are obvious.

However, there is fundamental problem in the model **a**: the singularity of the amplitude (22). The approach with finite time interval gives the finite result, which justifies the models **a** and **b** at least in principle. Nevertheless, the time-dependence $W_a \sim t^2$ and limit (24) seem very unusual. The corresponding calculation contains too many new elements. Due to this we view the results of the model **a** with certain caution. Besides, due to the zero momentum transfer in the $n\bar{n}$ -transition vertex, the model is extremely sensitive to the Σ . The process under study is *unstable*. The small change of antineutron self-energy $\Sigma = 0 \rightarrow \Sigma = V \neq 0$, or, similarly, effective momentum transfer in the $n\bar{n}$ transition vertex converts the model **a** to the model **b**: $W_a \rightarrow W_b$ with $W_b \ll W_a$. (This is because the process amplitude is in the peculiar point (see (22)) owing to zero momentum transfer. For the processes with non-zero momentum transfer the result is little affected by small change of q .) Although we don't see the specific reasons for above-mentioned scenario, it must not be ruled out. This is a point of great nicety.

Finally, the values $\tau^b = 1.2 \cdot 10^9$ s and $\tau^a = 10^{16}$ yr are interpreted as the estimations from below (conservative limit) and from above, respectively. So the realistic limit τ can be in the range

$$10^{16} \text{ yr} > \tau > 1.2 \cdot 10^9 \text{ s.} \quad (38)$$

Recall that for the free-space ab oscillations the ab transition probability is extremely sensitive to the difference of masses $m_a - m_b$ as well. For the neutron in the bound state ($n\bar{n}$ transition in finite nucleus) the result is the same [29], what is consistent with the qualitative process

picture: two-step process (dynamic $n\bar{n}$ conversion [4], annihilation) with the characteristic time $\tau_{ch} \sim 1/\Gamma$. The fact that process amplitude is in the peculiar point (unremovable peculiarity) is the basic reason why the range (38) is very wide. The estimation from below $\tau^b = 1.2 \cdot 10^9$ s exceeds the restriction given by the Grenoble reactor experiment [30] by a factor of 14 and the lower limit given by potential model by a factor of 5. At the same time the range of uncertainty of τ is too wide. Further theoretical and experimental investigations are desirable.

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